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Pareto efficiency in synthesizing shared autonomy policies with temporal logic constraints

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Abstract— **For systems in which control authority is shared by an autonomous controller and a human operator, it is important to find solutions that achieve a desirable system performance with a reasonable workload for the human operator. We formulate a shared autonomy system capable of capturing the interaction and switching control between an autonomous controller and a human operator, as well as the evolution of the operator's cognitive state in the working environment. To trade-off human's effort and the performance level, e.g., measured by the probability of satisfying the underlying temporal logic specification, a two-stage policy synthesis algorithm is proposed for generating Pareto efficient coordination and control policies with respect to user specified weights.**

I. Introduction

Despite the rapid progress in designing fully autonomous systems, many systems still require human's expertise to handle tasks which autonomous controllers cannot handle or which they have poor performance. Therefore, shared autonomy systems have been developed to bridge the gap between fully autonomous and fully human operated systems. In this paper, we examine a class of shared autonomy systems, featured by switching control between a human operator and an autonomous controller to collectively achieve a given control objective. Examples of such shared autonomy systems include robotic mobile manipulation [1], remote teleoperated mobile robots [2], human-in-the-loop autonomous driving vehicle [3], [4]. In particular, we consider control under temporal logic specifications.

One major challenge for designing shared autonomy policies under temporal logic specifications is making trade-offs between two possibly competing objectives: Achieving the optimal performance for satisfying temporal logic constraints and minimizing the operator's effort. Moreover, the operator's cognition is an inseparable factor in synthesizing shared autonomy systems since it directly influences the operator's performance, for example, a operator may have limited time span of attention and possible delays in response to a request. Although finding an accurate model of operator's cognition is an ongoing challenging topic within cognitive science, Markov models have been proposed to model and predict human operators' behaviors in various decision making tasks [5]–[7]. Adopting this modeling paradigm for the operator's cognition, we propose a formalism for shared autonomy systems capturing three important components: The operator, the autonomous controller and the cognitive model of the operator, into a stochastic *shared-autonomy system*. Precisely, the three components includes a Markov model representing the fully-autonomous system, a Markov model for the fully human-operated system, and a Markov model representing the evolution of operator's cognitive states under requests from the autonomous controller to the operator, or other external events. The uncertainty in the composed system comes from the stochastic nature of the underlying dynamical system and its environment as well as the inherent uncertainty in the operator's cognition. Switching from the autonomous controller to the operator can occur only at a particular set of human's cognitive states, influenced by requests from the autonomous controller to the operator, such as, pay more attention, be prepared for a possible future control action.

Under this mathematical formulation, we transform the problem of synthesizing a shared autonomy policy that coordinates the operator and the autonomous controller into solving a multi-objective Markov decision process (multi-objective MDP) with temporal logic constraints: One objective is to optimize the probability of satisfying the given temporal logic formula, and another objective is to minimize the human's effort over an infinite horizon, measured by a given cost function. The trade-off between multiple objectives is then made through computing the Pareto optimal set. Given a policy in this set, there is no other policy that can make it better for one objective than this policy without making it worse for another objective. In literature, Pareto optimal policies for multi-objective MDPs have been studied for the cases of long-run discounted and average rewards [8], [9]. The authors in [10] proposed the weighted-sum method for multi-objective MDPs with multiple temporal logic constraints by solving Pareto optimal policies for undiscounted time-bounded reachability or accumulated rewards. These aforementioned methods are not directly applicable in our problem due to the time unboundness in both satisfying these temporal logic constraints and the accumulated cost/reward. To this end, we develop a novel two-stage optimization method to handle the multiple objectives and adopt the so-called *Tchebychev scalarization method* [11] for finding a uniform coverage of all Pareto optimal points in the policy space. Finally, we conclude the paper with an algorithm that generates a Pareto-optimal policy achieving the desired trade-off from

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user-defined weights for coordinating the switching control between an operator and an autonomous controller for a stochastic system with temporal logic constraints.

II. Preliminaries

A vector in $\mathbb{R}^n$ is denoted $\vec{v} = (v_1, v_2, \ldots, v_n)$ where $v_i, 1 \leq i \leq n$ are the components of $\vec{v}$. Let the projection of the i-th component be $\pi_i(v) = v_i$ for $\vec{v}$. We denote the set of probability distributions on a set S by $\mathcal{D}(S)$. Given a probability distribution $D : S \to [0,1]$, let $\mathsf{Support}(D(s)) = \{s \in S \mid D(s) \neq 0\}$ be the set of elements with non-zero probabilities in D.

A. Markov decision processes and control policies

Definition 1: A *labeled Markov decision process (MDP)* is a tuple $M = \langle S, \Sigma, D_0, T, \mathcal{AP}, L, r, \gamma \rangle$ where S and Σ are finite state and action sets. $D_0 : S \to \mathbb{R}$ is the initial probability distribution over states. The transition probability function $T : S \times \Sigma \times S \to [0,1]$ is defined such that given a state $s \in S$ and an action $\sigma \in \Sigma$, $T(s, \sigma, s')$ gives the probability of reaching the next state s'. $\mathcal{AP}$ is a finite set of atomic propositions and $L : S \to 2^{\mathcal{AP}}$ is a labeling function. For a state s, $L(s)$ is a set of atomic propositions that are valid at state s. $r : S \times \Sigma \times S \to \mathbb{R}$ is a reward function giving the immediate reward $r(s, a, s')$ for reaching the state s' after taking action a at the state s and $\gamma \in (0,1)$ is the reward discount factor.

In this context, $T(s,a)$ gives a probability distribution over the set of states. $T(s,a)(s')$ and $T(s,a,s')$ both express the transition probability from state s to state s' under action a in M. A *path* is an infinite sequence $s_0 s_1 \ldots$ of states such that for all $i \geq 0$, there exists $a \in \Sigma$, $T(s_i, a, s_j) \neq 0$. We denote $\Gamma(s) \subseteq \Sigma$ to be a set of actions enabled at the state s. That is, for each $a \in \Gamma(s)$, $\mathsf{Support}(T(s,a)) \neq \emptyset$.

A *randomized policy* in M is a function $f : S^* \to \mathcal{D}(\Sigma)$ that maps a finite path into a probability distribution over actions. Given a policy f, for a measurable function ϕ that maps paths into reals, we write $\mathbb{E}^f_s[\phi]$ (resp. $\mathbb{E}^f_{D_0}[\phi]$) for the expected value of ϕ when the MDP starts in state s (resp. an initial distribution of states D_0) and the policy f is used. A policy f induces a probability distribution over paths in M. The state reached at step t is a random variable X_t and the action being taken at state X_t is also a random variable, denoted A_t.

B. Synthesis for MDPs with temporal logic constraints

We use linear temporal logic (LTL) [13] to specify a set of desired system properties such as safety, liveness, persistence and stability. In the following, we present some basic preliminaries for LTL specifications and introduce a product operation for synthesizing policies in MDPs under such specifications.

A formula in LTL is built from a finite set of atomic propositions $\mathcal{AP}$, true, false and the Boolean and temporal connectives $\wedge, \vee, \neg, \Rightarrow, \Leftrightarrow$ and $\Box$ (always), $\mathcal{U}$ (until), $\Diamond$ (eventually), $\bigcirc$ (next). Given an LTL formula φ as the system specification, one can always represent it by a deterministic Rabin automaton (DRA) $\mathcal{A}_\varphi = \langle Q, 2^{\mathcal{AP}}, \delta, I, \mathsf{Acc} \rangle$ where Q is a finite state set, $2^{\mathcal{AP}}$ is the alphabet, $I \in Q$ is the initial state, and $\delta : Q \times 2^{\mathcal{AP}} \to Q$ is the transition function. The acceptance condition Acc is a set of tuples $\{(J_i, K_i) \in 2^Q \times 2^Q \mid i = 0, 1, \ldots, m\}$. The run for an infinite word $w = w[0]w[1]\ldots \in (2^{\mathcal{AP}})^\omega$ is an infinite sequence of states $q_0 q_1 \ldots \in Q^\omega$ where $q_0 = I$ and $q_{i+1} = \delta(q_i, w[i])$. A run $\rho = q_0 q_1 \ldots$ is accepted in $\mathcal{A}_\varphi$ if there exists at least one pair $(J_i, K_i) \in \mathsf{Acc}$ such that $\mathsf{Inf}(\rho) \cap J_i = \emptyset$ and $\mathsf{Inf}(\rho) \cap K_i \neq \emptyset$ where $\mathsf{Inf}(\rho)$ is the set of states that appear infinitely often in ρ.

We define a product operation between a labeled MDP and a DRA.

Definition 2: Given a labeled MDP $M = \langle S, \Sigma, D_0, T, \mathcal{AP}, L, r, \gamma \rangle$ and a DRA $\mathcal{A}_\varphi = \langle Q, 2^{\mathcal{AP}}, \delta, I, \{(J_i, K_i) \mid i = 1, \ldots, m\} \rangle$, the *product MDP* is $\mathcal{M} = M \ltimes \mathcal{A}_\varphi = \langle V, \Sigma, \Delta, \boldsymbol{D}_0, \boldsymbol{r}, \gamma, \mathsf{Acc} \rangle$, with components defined as follows: $V = S \times Q$ is the set of states. Σ is the set of actions. $\boldsymbol{D}_0 : V \to [0,1]$ is the initial distribution, defined by $\boldsymbol{D}_0((s,q)) = D_0(s)$ where $q = \delta(I, L(s))$. $\Delta : V \times \Sigma \times V \to [0,1]$ is the transition probability function. Given $v = (s,q)$, σ, $v' = (s', q')$ and $q' = \delta(q, L(s'))$, let $\Delta(v, \sigma, v') = T(s, \sigma, s')$. The reward function is defined as $\boldsymbol{r} : V \times \Sigma \times V \to \mathbb{R}$ where given $v = (s,q)$, $v' = (s', q')$, $a \in \Sigma$, $\boldsymbol{r}(v, a, v') = r(s, a, s')$. The acceptance condition is $\mathsf{Acc} = \{(\hat{J}_i, \hat{K}_i) \mid \hat{J}_i = S \times J_i, \hat{K}_i = S \times K_i, i = 1, \ldots, m\}$.

By construction, the total (discounted) rewards of a path $\rho = v_0 v_1 \ldots \in V^\omega$ equal the total (discounted) rewards of the path $\rho' = s_0 s_1 \ldots \in S^\omega$ with $s_i = \pi_1(v_i)$ for all $i \geq 0$. The problem of maximizing the probability of satisfying the LTL formula φ in M is transformed into a problem of maximizing the probability of reaching a particular set in the product MDP $\mathcal{M}$, which is defined next.

Definition 3: [14] An *end component* for the product MDP $\mathcal{M}$ is a pair (W, f) where $W \subseteq V$ is a non-empty set of states and $f : W \to \mathcal{D}(\Sigma)$ is a randomized policy. Moreover, the policy f is defined such that for any $v \in W$, for any $a \in \mathsf{Support}(f(v))$, $\sum_{v' \in W} \Delta(v, a, v') = 1$; and the induced directed graph $(W, \to_f)$ is strongly connected. Here, $v \to_f v'$ is an edge in the directed graph if $\Delta(v, a, v') > 0$ for some $a \in \mathsf{Support}(f(v))$. An accepting end component (AEC) is an end component such that $W \cap \hat{J}_i = \emptyset$ and $W \cap \hat{K}_i \neq \emptyset$ for some $i \in \{1, \ldots, m\}$.

Let the set of AECs in $\mathcal{M}$ be denoted $\mathsf{AEC}(\mathcal{M})$ and the set of *accepting end states* be denoted by $\mathcal{W} = \{v \mid \exists (W, f) \in \mathsf{AEC}(\mathcal{M}), v \in W\}$. Note that, by definition, for each AEC (W, f), by exercising the associated policy f, the probability of reaching any state in W is 1. Due to this property, once we enter some state $v \in \mathcal{W}$, we can find at least one accepting end component (W, f) such that $v \in W$, and initiate the policy f such that for some $i \in \{1, \ldots, m\}$, all states in $\hat{J}_i$ will be visited only a finite number of times and some state in $\hat{K}_i$ will be visited infinitely often.

III. Modeling Human-in-the-loop stochastic system

We aim to synthesize a shared autonomy policy that switches control between a human operator and an autonomous controller. The stochastic system controlled by the operator and the autonomous controller, gives rise to two different MDPs with the same set of states S, the same set $\mathcal{AP}$ of atomic propositions and the same labeling function $L : S \to 2^{\mathcal{AP}}$, but possibly different sets of actions and transition probability functions.

- Autonomous controller: $M_A = \langle S, \Sigma_A, T_A, \mathcal{AP}, L\rangle$ where $T_A : S \times \Sigma_A \times S \to [0, 1]$ is the transition probability function of the system when controlled by the autonomous controller.
- Human operator: $M_H = \langle S, \Sigma_H, T_H, \mathcal{AP}, L\rangle$ where $T_H : S \times \Sigma_H \times S \to [0, 1]$ is the transition probability function of the system when controlled by the operator.

Let $D_0^M : S \to [0, 1]$ be the initial distribution of states, same for both M_A and M_H. The models M_A and M_H can be constructed either from prior knowledge or from experiments by applying a policy that samples each action from each state a sufficient amount of times [16]. In the following context, we refer to the autonomous controller simply as the controller.

In the shared autonomy system, the interaction between the controller and the operator is often made through a dialogue system [17]. The controller may send a request of attention, or some other signal to the operator. The operator may grant the request, or respond to signals, depending on his current workload, level of attention. Admitting that it is not possible to capture all aspects of an operator's cognitive states, we have the following model to capture the evolution of the modeled cognitive state.

Definition 4: The operator's cognition in the shared autonomy system is modeled as an MDP

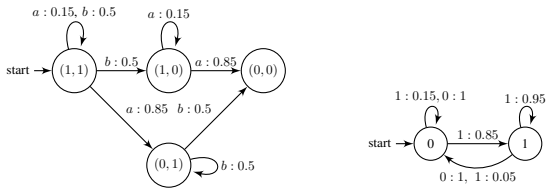
$$M_C = \langle H, E, D_0^H, T_C, C, \gamma, H_s\rangle$$

where H represents a finite set of cognitive states. E is a finite set of events that trigger changes in cognitive state. $D_0^H : H \to [0, 1]$ is the initial distribution. $T_C : H \times E \times H \to [0, 1]$ is the transition probability function. $C : H \times E \times H \to \mathbb{R}$ is the cost function. $C(h, e, h')$ is the cost of human effort for the transition from h to h' under event e. $\gamma \in (0, 1)$ is the discount factor. $H_s \subseteq H$ is a subset of states at which the operator can take over control.

This cognitive model can be generalized to accomodate different model of operator's interaction with the controller. The set E of events can be requests sent by the controller to the operator, a workload that the operator assigns to himself, or any other external event that influences the operator's cognitive state. This model generalizes the model of operator's cognition in [18], in which an event is a request to increase, decrease, or maintain the operator's attention in the control task. In particular, it is assumed that in a particular set of states, transitions from the controller to the operator can happen. For instance, for tele-operated robotic arm or semi-autonomous vehicle, operator may take over control only when he is aware of the system's state and not occupied by other tasks [17]. The model is flexible and can be extended to other cognitive models in shared autonomy. In this paper, we assume the model of operator is given. One can obtain such a model by learning from experimental data.

We illustrate the concepts using the robotic manipulation task.

Example 1: Consider a robot manipulator having to pick up the objects on a table and place them into a box. There are two types of objects, small and large. For small and large objects, the probabilities of a successful pick-and-place maneuver performed by the controller is 0.85 and 0.5 respectively. The MDP for the controller is shown in Figure 1a. With an operator tele-operating the robot, the probabilities of a successful pick-and-place maneuver is 0.95 and 0.75 respectively. The operator's cognitive model includes two cognitive states: 0 represents the state when the operator does not pay any attention to the system (at the attention level 0), and 1 represents the state when he pays full attention (at the attention level 1). The set of events in M_C is the requests of the operator's attention to the task, $E = \{0, 1\}$ where $e \in E$ represents the current requested attention level is e. For any $h \in \{0, 1\}$, $e \in E$, let $C(h, e, h') = 10$ for $h' = 1$, otherwise 5. The transition probability function of M_C is shown in Figure 1b.



(a) (b)

Fig. 1: (a) The MDP for the robotic arm controlled by the controller. A state (n, m) represents there are n small objects and m large objects remaining to be picked. The available actions are a and b for picking up small and large objects, respectively. The MDP for the robotic manipulator tele-operated by the operator can be obtained by changing the probabilities on the transitions. (b) The MDP M_C for modeling the operator's cognition.

Given two MDPs, M_A for the controller and M_H for the operator, and an operator's cognitive model M_C, we construct a *shared autonomy stochastic system* as follows.

$$M_{SA} = \langle \boldsymbol{S}, \Sigma, \boldsymbol{T}, D_0, \mathcal{AP}, L, \mathsf{Cost}, \gamma\rangle$$

where $\boldsymbol{S} = S \times H$ is the set of states. A state (s, h) includes a state s of the system and a cognitive state h of the operator. $\Sigma = (\Sigma_A \cup \Sigma_H) \times E$ is the set of actions. If $(a, e) \in \Sigma_A \times E$, the system is controlled by the controller and the event affecting the operator's cognition is e. If $(a, e) \in \Sigma_H \times E$, the system is controlled by the operator and the event affecting the operator's cognition is e. $\boldsymbol{T} : \boldsymbol{S} \times \Sigma \times \boldsymbol{S} \to [0, 1]$ is the transition probability function,

defined as follows. Given a state (s,h) and action $(a,e) \in \Sigma_A \times E$, $\boldsymbol{T}((s,h),(a,e),(s',h')) = T_A(s,a,s')T_C(h,e,h')$, which expresses that the controller acts and triggers an event that affects the operator's cognitive state. Given a state (s,h) for $h \in H_s$, and action $(a,e) \in \Sigma_H \times E$, $\boldsymbol{T}((s,h),(a,e),(s',h')) = T_H(s,a,s')T_C(h,e,h')$, which expresses that the operator controls the system and an event e happens and affects the cognitive state. $D_0 : S \times H \to [0,1]$ is the initial distribution. $D_0(s,h) = D_0^M(s) \times D_0^H(h)$, for all $s \in S$, $h \in H$. $L : \boldsymbol{S} \to 2^{\mathcal{AP}}$ is the labeling function such that $L((s,h)) = L(s)$. $\mathsf{Cost} : \boldsymbol{S} \times \Sigma \times \boldsymbol{S} \to R$ is a cost function for operator's effort defined over the state and action spaces and $\mathsf{Cost}((s,h),(a,e),(s',h')) = C(h,e,h')$. $\gamma \in (0,1)$ is the discount factor, the same in M_C. Slightly abusing the notation, we denote the labeling function in M_{SA} the same as the labeling function in M.

Example 1: (Cont.) We construct MDP M_{SA} in Figure 2 for the robotic arm example. For example, $\boldsymbol{T}((1,1),0),(a_A,1),((0,1),1)) = T_A((1,1),a_A,(0,1)) \cdot T_C(0,1,1) = 0.75 \cdot 0.85 = 0.7225$, which means the probability of the robot successfully picking up a small object and placing it into the box while the operator changes his cognitive state to 1 (fully focused) upon the robot's request is 0.7225. Also it is noted that from the states $((0,1),0)$ and $((1,1),0)$, no action of the operator is enabled. The cost function is defined such that $\mathsf{Cost}((q,h),a,(q,h')) = 10$ if $h' = 1$, otherwise 5.

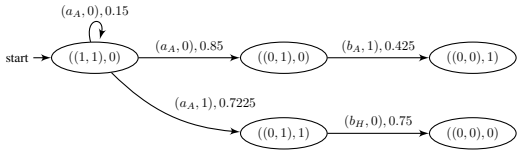


Fig. 2: A fragment of MDP M_{SA} for robotic arm example (note only a subset of states and transitions are shown). Subscripts A and H distinguish actions performed by the controller (A) and the operator (H), respectively.

The main problem we solve is the following.

Problem 1: Given a stochastic system under control switching between an operator and an autonomous controller, modeled as MDPs M_H and M_A, a model of the operator's cognition M_C, and an LTL specification φ, compute a policy that is Pareto optimal (defined in section IV-A) with respect to two objectives: 1) Maximizing the discounted probability of satisfying the LTL specification φ and 2) minimizing the discounted total cost of the operator's effort over an infinite horizon.

By following a Pareto optimal policy, we achieve a balance between two objectives: It is impossible to make one better off without making the other one worse off.

IV. Synthesis for shared autonomy policy

Given an MDP $M_{SA} = \langle \boldsymbol{S}, \Sigma, \boldsymbol{T}, D_0, \mathcal{AP}, L, \mathsf{Cost}, \gamma \rangle$ and a DRA $\mathcal{A}_\varphi = \langle Q, 2^{\mathcal{AP}}, \delta, I, \{(J_i, K_i) \mid i = 1, \ldots, m\} \rangle$, the product MDP following Definition 2 is $\mathcal{M} = M_{SA} \ltimes \mathcal{A}_\varphi = \langle V, \Sigma, \Delta, \boldsymbol{D}_0, \mathsf{Cost}^{\mathcal{M}}, \gamma, \mathsf{Acc} \rangle$. The policy maximizing the probability of satisfying the LTL specification is obtained by first computing the set of AECs in $\mathcal{M}$ and then finding a policy that maximizing the probability of hitting the set $\mathcal{W}$ of states contained in AECs (see Section II-B).

For quantitative LTL objectives, for example, maximizing the probability of satisfying an LTL formula, or a discounted reward objective over an infinite horizon, a memoryless policy in the product MDP suffices for optimality [19], [20]. In the following, by policies, we mean memoryless ones in the product MDP.

Problem 1 is in fact a multi-objective optimization problem for which we need to balance the cost of the operator's effort and the probability of satisfying LTL constraints. However, the solutions for multi-objective MDPs cannot be directly applied due to the constraint that once the system runs into an AEC of $\mathcal{M}$, the policy should be constrained such that all states in that AEC are visited infinitely often. Based on the particular constraint, we divide the original problem into a two-stage optimization problem: The policy synthesis for AECs is separated from solving a multi-objective MDP formulated before reaching a state in an AEC.

A. Pareto efficiency before reaching the AECs

The first stage is to balance between a quantitative criterion for a temporal logic objective and a criterion with respect to the cost of the operator's effort before a state in the set $\mathcal{W}$ is reached. Remind that $\mathcal{W}$ is the union of states in the accepting end components of $\mathcal{M}$. We formulate it as an multi-objective MDP. However, for objectives of different types, such as, discounted, undiscounted, and limit-average. the scalarization method for solving multi-objective MDPs does not apply. Thus, we consider to use the discounted reachability property [21] for the given LTL specification, as well as discounted costs for the operator's attention, with the same discount factor $\gamma \in (0,1)$ specifying the relative importance of immediate rewards.

For an LTL specification, discounting in the state sequence before reaching the set $\mathcal{W}$ means that the number of steps for reaching $\mathcal{W}$ is concerned [21]. With discounting though, a policy has smaller expected number of steps in reaching $\mathcal{W}$ is considered to be better than the other, which has the same probability of reaching the set $\mathcal{W}$.

The reward function $r_1 : V \times \Sigma \times V \to \{0,1\}$ is defined such that $r_1(v,a,v') = 1$ if and only if $v \notin \mathcal{W}$ and $v' \in \mathcal{W}$. Otherwise $r_1(v,a,v') = 0$. Given a set $\mathcal{W}$ of states, let $U^*_{\mathsf{AEC}} : \mathcal{W} \to \mathbb{R}$ be a function that maps each state $v \in \mathcal{W}$ to the discounted accumulated cost $U^*_{\mathsf{AEC}}(v)$ of the operator's effort, which is required for the system starting from v to remain in an AEC under the optimal policy for the second stage. The reward function $r_2 : V \times \Sigma \times V \to \mathbb{R}$ is defined such that: If $v \notin \mathcal{W}$ and $v' \notin \mathcal{W}$, let $r_2(v,a,v') = -\mathsf{Cost}^{\mathcal{M}}(v,a,v')$. Else, if $v \notin W$ and $v' \in \mathcal{W}$, let $r_2(v,a,v') = -U^*_{\mathsf{AEC}}(v')$. Otherwise, let $r_2(v,a,v') = 0$. We denote $\vec{r} = (r_1, r_2)$ as the vector of

reward functions. The function $U^*_{\mathsf{AEC}} : \mathcal{W} \to \mathbb{R}$ is computed in the next section.

Next, we modify the transition probability function Δ of $\mathcal{M}$ as follows: If $v \in \mathcal{W}$, for all $a \in \Gamma(v)$, let $\hat{\Delta}(v, a, v) = 1$. Otherwise $v \in V \setminus \mathcal{W}$, for all $a \in \Gamma(v)$, $\hat{\Delta}(v, a) = \Delta(v, a)$. The product MDP with the modified transition probability function is denoted as $\hat{\mathcal{M}}$.

Definition 5: Given the product MDP $\hat{\mathcal{M}}$ with a modified transition probability function, for a state $v \in V \setminus \mathcal{W}$, by definitions of the reward functions, the *discounted probability* for reaching the set $\mathcal{W}$ under policy $f : V \setminus \mathcal{W} \to \mathcal{D}(\Sigma)$ is $U_1(v, f) = \mathbb{E}^f_v \left[\sum_{t=0}^{\infty} \gamma^t \cdot r_1(X_t, A_t, X_{t+1})\right]$. The discounted total reward with respect to the operator's effort for a policy $f : V \setminus \mathcal{W} \to \mathcal{D}(\Sigma)$ and a state v is $U_2(v, f) = \mathbb{E}^f_v \left[\sum_{t=0}^{\infty} \gamma^t \cdot r_2(X_t, A_t, X_{t+1})\right]$.

The *discounted value profile*, at v for policy f, is defined as $\vec{U}(v, f) = (U_1(v, f), U_2(v, f))$.

Definition 6: [8] Given an MDP $\mathcal{M} = \langle V, \Sigma, \Delta\rangle$ and a vector of reward functions $\vec{r} = (r_1, r_2, \ldots, r_n)$, for a given state $v \in V$, policy f *Pareto-dominates* policy f' at state v if and only if $\vec{U}(v, f) = (U_1(v, f), \ldots, U_n(v, f)) \neq \vec{U}(v, f') = (U_1(v, f'), \ldots, U_n(v, f'))$ and for all $i = 1, \ldots, n, U_i(v, f) \geq U_i(v, f')$. A policy f is *Pareto optimal* in a state $v \in V$ if there is no other policy f' Pareto-dominating f. For a Pareto-optimal policy f at state v, the corresponding value profile $\vec{U}(v, f)$ is referred to as a *Pareto-optimal point (or an efficient point)*. The set of Pareto-optimal point are called the *Pareto set*.

A Pareto optimal policy f for a given initial distribution is defined analogously by comparing the expectations of value functions under the initial distribution.

We employ Tchebycheff scalarization method [11], [22] to find Pareto optimal policies for user specified weights. First, we solve a set of single objective MDPs, one for each reward function. Let $U_i(\cdot, f^*_i) : V \to \mathbb{R}$ be the value function of the optimal policy f^*_i with respect to the i-th reward function. Let $U^I_i = \sum_{v \in V} \boldsymbol{D}_0(v) U_i(v, f^*_i)$ for $i = 1, 2$. Given a weight vector $\vec{w} = (w_1, w_2)$ where w_i is the weight for the i-th criterion such that $w_1 + w_2 = 1$, a Pareto optimal policy associated with the weight vector $\vec{w}$ can be found with the following nonlinear program:

$$\begin{aligned}
&\min_{x} \max_{i=1,2} (\lambda_i \cdot (U^I_i - R_i \cdot x)) + \epsilon \sum_{i=1,2} \lambda_i \cdot \left(U^I_i - R_i \cdot x\right) \\
&\text{subject to: } \forall v \in V \setminus \mathcal{W}, \\
&\sum_{a \in \Gamma(v)} x(v, a) = \boldsymbol{D}_0(v) + \gamma \sum_{v' \in V} \sum_{a' \in \Gamma(v')} \Delta(v', a', v) \cdot x(v', a'), \\
&\text{and } \forall v \in V \setminus \mathcal{W}, \forall a \in \Sigma, \quad x(v, a) \geq 0,
\end{aligned} \tag{1}$$

where ϵ is a small positive real that can be chosen arbitrarily, $x(v, a)$ is interpreted as the expected discounted frequency of reaching the state v and then choosing action a, $R_i \cdot x = \sum_{v \in V} \sum_{a \in \Gamma(v)} \sum_{v' \in V} r_i(v, a, v') \Delta(v, a, v') x(v, a)$, and $\vec{\lambda}$ is a positive weighting vector computed from a weight $\vec{w}$, the ideal points and the Nadir points [22] for all reward functions (detailed in Appendix). The nonlinear programming problem can then be formulated into a linear programming problem in the standard way by setting a new variable $z = \max_{i=1,2}(\lambda_i \cdot (U^I_i - R_i \cdot x))$. The Pareto optimal policy $f : V \to \mathcal{D}(\Sigma)$ is defined such that

$$f(v)(a) = \frac{x(v, a)}{\sum_{a \in \Gamma(v)} x(v)}, \tag{2}$$

which selects action a with probability $f(v)(a)$ from the state v, for all $v \in V$, $a \in \Gamma(v)$.

Example 2: Continue with the robot manipulation task. Given the discount factor $\gamma = 0.98$, for the first objective: As quickly as possible of reaching a state at which all objects are in the box, the optimal strategy f^*_1 is shown in the first row of Table I. Intuitively, the robot starts by requesting the operator to increase his level of attention and wants to switch control to the operator as soon as possible as the latter has higher probability of success for a pick-and-place maneuver. Alternatively, the optimal policy with respect to minimizing the cost of the operator's effort, is to let the robot pick up all the objects since by doing so, eventually all the objects will be collected into the box. The strategy f^*_2 is shown in the second row of Table I.

Now suppose that a user gives a weight vector $(0.8, 0.2)$, we compute the normalized weight vector $\vec{\lambda} = (11.93, 0.02)$, with the method in the Appendix. By solving the linear programming problem in (1), we obtain a Pareto-optimal policy f^*_P shown in the third row of Table I. Note that when it comes to the small object, if the operator's current attention is high, the robot will request him to decrease his attention level. Therefore, if the object fails to be picked up through tele-operation, the autonomous controller will take over for picking up the small object.

Figure 3 shows the state value for the initial state $v_0 = ((1, 1), 0)$ with respect to reward functions r_1, r_2, under the policies f^*_1, f^*_2 and a subset of Pareto optimal policies, one for each weight vector $\vec{w}$ in the set $\{(\beta, 1-\beta) \mid \beta = \frac{k}{10}, k = 1, 2, \ldots, 9\}$.

TABLE I: Policies for pick-and-place task

States:	$((1,1),0)$	$((1,1),1)$	$((1,0),0)$	$((1,0),1)$	$((0,1),1)$	$((0,1),0)$
f^*_1:	$(a_A, 1)$	$(b_H, 1)$	$(a_A, 1)$	$(a_H, 1)$	$(b_H, 1)$	$(b_A, 1)$
f^*_2 :	$(a_A, 0)$	NA	$(a_A, 0)$	NA	NA	$(b_A, 0)$
f^*_P	$(a_A, 1)$	$(b_H, 1)$	$(a_A, 0)$	$(a_H, 0)$	$(b_H, 1)$	$(b_A, 1)$

So far we have introduced a method for synthesizing Pareto optimal policies before reaching a state in one of the accepting end components. Next, we introduce a constrained optimization for synthesizing a policy that minimize the expected discounted cost of staying in an AEC and visiting all the states in that AEC infinitely often.

B. A constrained optimization for accepting end components

For a state v in $\mathcal{W}$, one can identify at least one AEC (W, f) such that $v \in W$. It is noted that the policy $f : V \to \mathcal{D}(\Sigma)$ is a randomized policy that ensures every state in W is visited infinitely often with probability 1 [15]. However, there might be more than one AEC that contains a state

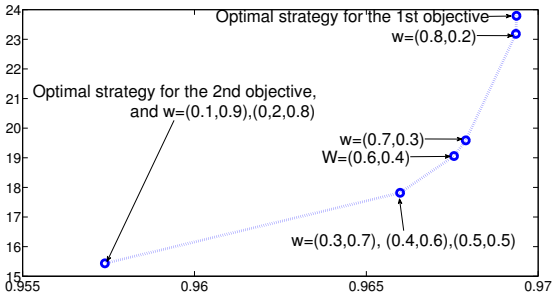


Fig. 3: The state values of the initial state with respect to reward functions r_1, r_2, under policies f_1^*, f_2^* and a set of Pareto optimal policies f_P^*, one for each weight vectors in the set $\{(\beta, 1-\beta) \mid \beta = \frac{k}{10}, k = 1, 2, \ldots, 9\}$. The x-axis and y-axis represent the values of the initial state under the 1st and 2nd criteria, respectively.

v, and we need to decide which AEC to stay in such that the expected discounted cost of the operator's effort for the control execution over an infinite horizon is minimized.

We consider a constrained optimization problem: For each AEC (W, f) where $W \subseteq V$ and $f : W \to \mathcal{D}(\Sigma)$, solve for a policy $g : W \to \mathcal{D}(\Sigma)$ such that the cost of the operator's effort for staying in that AEC is minimized. The constrained optimization problem is formulated as follows.

$$\begin{aligned} &\min_g U_{\mathsf{AEC}}(v, g, W) = \sum_{k=0}^{\infty} \gamma^k \cdot \mathbb{E}_v^g[\mathsf{Cost}^{\mathcal{M}}(X_t, A_t, X_{t+1})] \\ &\text{subject to:} \\ &\Pr^g(\forall t, \exists t' > t, X_{t'} = v) = 1, \forall v \in W, \\ &\Pr^g(\exists t, X_t = v) = 0, \forall v \in V \setminus W, \\ &g(v)(a) = 0, \forall v \in W, \forall a \notin \Gamma(v), \end{aligned} \tag{3}$$

where the term $\Pr^g(\forall t, \exists t' > t, X_{t'} = v)$ measures the probability of infinitely revisiting state v under policy g.

The linear program formulated for solving (3) can be obtained as follows:

$$\begin{aligned} &\min_x \sum_{v \in V} \sum_{a \in \Gamma(v)} \left[x(v,a) \cdot \left(\sum_{v' \in V} \mathsf{Cost}^{\mathcal{M}}(v,a,v') \Delta(v,a,v') \right) \right] \\ &\text{subject to: for } v \in W, \\ &\sum_{a \in \Gamma(v)} x(v,a) = \eta(v) + \gamma \sum_{v' \in V} \sum_{a' \in \Gamma(v')} \Delta(v', a', v) \cdot x(v', a'), \\ &\forall v \in W, \forall a \in \Sigma, x(v,a) \geq 0, \\ &\forall v \in W, \sum_{a \in \Gamma(v)} x(v,a) \geq \varepsilon, \forall a \notin \Gamma(v), x(v,a) = 0, \text{ and} \\ &\sum_{a \in \Gamma(v)} x(v,a) = 0, \forall v \in V \setminus W, \end{aligned} \tag{4}$$

where ε is a pre-defined arbitrarily small positive real. $\eta : W \to [0, 1]$ is the initial distribution of states when entering the set W. Because for single objective optimization the optimal state value does not depend on the initial distribution [23], η can be chosen arbitrarily from the set of distributions over W. The physical meaning of $\sum_{a \in \Gamma(v)} x(v, a)$ is the discounted frequency of visiting the state v, which is strictly smaller than the frequency of visiting the state v as long as $\gamma \neq 1$. By enforcing the constraints $\sum_{a \in \Gamma(v)} x(v, a) \geq \varepsilon$, we ensure that the frequency of visiting every state in W is non-zero, i.e., all states in W will be visited infinitely often.

The solution to (4) produces a memoryless policy $g^* : W \to \mathcal{D}(\Sigma)$ that chooses action a at a state v with probability $g^*(v)(a) = \frac{x(v,a)}{\sum_{a \in \Gamma(v)} x(v,a)}$. Using policy evaluation [24], the state value $U^*_{\mathsf{AEC}}(v, W)$ for each $v \in W$ under the optimal policy g^* can be computed. Then, the terminal cost $U^*_{\mathsf{AEC}} : \mathcal{W} \to \mathbb{R}$ is defined as follows.

$$U^*_{\mathsf{AEC}}(v) = \min_{(W,f) \in \mathsf{AEC}} U^*_{\mathsf{AEC}}(v, W)$$

and the policy after hitting the state v is g such that $U^g_{\mathsf{AEC}}(v, W) = U^*_{\mathsf{AEC}}(v, W) = U^*_{\mathsf{AEC}}(v)$.

We now present Algorithm 1 to conclude the two-state optimization procedure. A Pareto-optimal policy obtained with Algorithm 1 is exact.

Algorithm 1: `TwoStageOptimization`

Input: The MDP M_A, M_H and M_C, a specification automaton DRA $\mathcal{A}_\varphi$, and a weight $\vec{w}$.

Output: A pareto policy f for the discounted reachability and a partial function Policy : $V \to \mathcal{F}$, where $\mathcal{F}$ is the set of randomized policies. Policy(v) is the policy to follow after state v is reached.

begin

- $\mathcal{M}$ = `GetProductMDP` $(M_A, M_H, M_C, \mathcal{A}_\varphi)$;
- $\mathsf{AEC}(\mathcal{M})$ = `GetAEC` $(\mathcal{M})$; `/* Compute the accepting end components. */`
- **for** $(W, f) \in \mathsf{AEC}$ **do**
  - g^*_W =`ConstrainedOptAEC` $(W, \mathsf{Cost}^{\mathcal{M}})$; `/* Solve (4). */`
  - $U^*_{\mathsf{AEC}}(v, W)$ =`PolicyEvaluate` $(W, \mathsf{Cost}^{\mathcal{M}}, g^*_W)$;
- $\mathcal{W} = \cup_{(W,f) \in \mathsf{AEC}} W$;
- **for** $v \in \mathcal{W}$ **do**
  - $U^*_{\mathsf{AEC}}(v) = \min_{(W,f) \in \mathsf{AEC}} U^*_{\mathsf{AEC}}(v, W)$;
  - $\mathsf{Policy}(v) = g^*_W$ for which W such that $U^*_{\mathsf{AEC}}(v, W) = U^*_{\mathsf{AEC}}(v)$.
- $\vec{r}$ = `GetRewardVec` $(\mathcal{M}, \{U^*_{\mathsf{AEC}}(v) \mid v \in \mathcal{W}\}, \mathcal{W})$; `/* Formulate the reward vector according to Definition 5. */`
- $\hat{\mathcal{M}}$ =`ModifyTransitionProb` $(\mathcal{M}, \mathcal{W})$; `/* Modify the transition probability function of` $\mathcal{M}$. `*/`
- f =`GetParetoOptimal` $(\vec{r}, \hat{\mathcal{M}}, \vec{w})$ `/* Solve (1) and obtain the Pareto optimal policy` f `as in (2). */`
- **return** f, Policy.

C. Complexity

We now discuss the complexity of the algorithm. Starting with an MDP and a LTL formula φ, we first construct a DRA $\mathcal{A}_\varphi$ which in the worst case has size (number of states) doubly exponential in the size of the formula, and the time complexity is double exponential as well. The product MDP $\mathcal{M}$ is polynomial in the sizes of M_{SA} and $\mathcal{A}_\varphi$. Then the set of end components are identified in time polynomial in the size of the product [14], [15]. Linear programming problems can be solved in polynomial time in the number of variables and constraints. Here, we solve linear programming problems associated with the computation of optimal policies for each single objective, the constrained optimization problems in (4) for each AEC and the computation of a Pareto-optimal policy for a given weight vector. Overall, the algorithm is polynomial in $\mathcal{M}$ and double exponential in the size of the specification formula.

V. An example on shared autonomy

We apply Algorithm 1 to a robotic motion planning problem in a stochastic environment. Figure 4a shows a gridworld environment of four different terrains: Pavement, grass, gravel and sand. In each terrain, the mobile robot can move in four directions (heading north 'N', south 'S', east 'E', and west 'W'). There is onboard feedback controller that implements these four maneuvers. Using the onboard controller, the probability of arriving at the correct cell is 0.95 for pavement, 0.8 for grass, 0.75 for gravel and 0.65 for sand. Alternatively, if the robot is operated a human, it can implement the four actions with a better performance for terrains grass, sand and gravel. The probability of arriving at the correct cell under the human's operation is 0.95 for pavement, 0.9 for grass, 0.85 for gravel and 0.8 for sand. The objective is that either the robot has to visit region R_1 and then R_2, in this order, or it needs to visit region R_3 infinitely often, while avoiding all the obstacles. Formally, the specification is expressed with an LTL formula $\varphi = (\Diamond(R_1 \wedge \Diamond R_2) \vee \Box\Diamond R_3) \wedge \Box\Diamond\neg\text{Unsafe}$.

Figure 4b is the cognitive model of the operator, including three states : L, M and H represent that the operator pays low, moderate, and high attention to the system respectively. The costs of paying low, moderate and high attention to the system are 1, 5, and 10, respectively. Action '$+$' (resp. '$-$') means a request to increase (resp. decrease) the attention and action λ means a request to maintain the current attention. The operator takes over control at state H.

During control execution, we aim to design a policy that coordinates the switching of control between the operator and the autonomous controller, i.e., onboard software controller. The policy should be Pareto optimal in order to balance between maximizing the discounted probability of satisfying the LTL formula φ, and minimizing the expected discounted total cost of the operator's efforts. Figure 5 shows the state value for the initial state with respect to reward functions r_1 for the LTL formula and r_2 for the cost of the operator's effort, under the single objective optimal policy

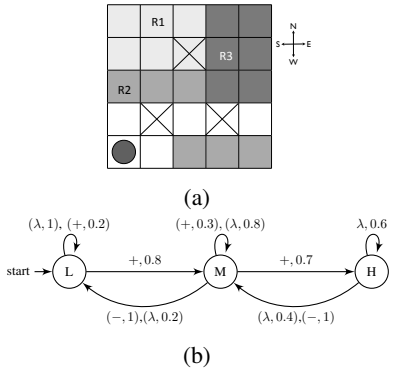


Fig. 4: (a) A 5×5 gridworld, where the disk represents the robot, the cells R_1, R_2, and R_3 are the interested regions, the crossed cells are obstacles. We assume that if the robot hits the wall (edges), it will be bounced back to the previous cell. Different grey scales represents different terrains: From the darkest to the lightest, these are sand, grass, pavement and gravel. (b) The MDP M_C of the operator.

f_1^* and f_2^*, and a subset of Pareto optimal policies, one for each weight vectors $\vec{w}$ in the set $\{(\beta, 1-\beta) \mid \beta = \frac{k}{10}, k = 1, 2, \ldots, 9\}$. The implementations are in Python and Matlab on a desktop with Intel(R) Core(TM) processor and 16 GB of memory. For each weight vector, the average time for computing a Pareto-optimal policy is ~ 600 seconds.

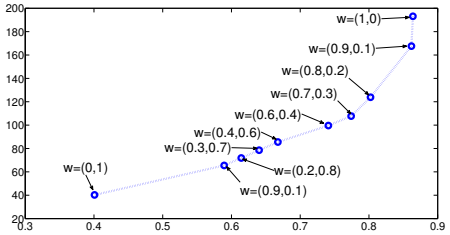


Fig. 5: The state values of the initial state given reward functions r_1, r_2, under policies f_1^*, f_2^* and a set of Pareto optimal policies f_P^*, for each $\vec{w} \in \{(\beta, 1-\beta) \mid \beta = \frac{k}{10}, k = 1, 2, \ldots, 9\}$. The x-axis represents the values of the initial state for discounted probability of satisfying the LTL specification. The y-axis represents the values of the initial state with respect to the cost of the operator's effort.

VI. Concluding remarks and critiques

We developed a modeling and synthesis framework for a class of shared autonomy systems featured by switching control between a human operator and an autonomous controller. In the presence of inherent uncertainties in the systems' dynamics and the evolution of the operator's cognitive states, we proposed a two-stage optimization method to trade-off the operator's effort for the system's performance in satisfying a given temporal logic specification. Moreover, the solution method can also be extended for solving multi-objective MDPs with temporal logic constraints. In the

following, we discuss some possible directions for future work.

We employed two MDPs for modeling the system operated by the operator and for representing the evolution of cognitive states triggered by external events such as workload, fatigue and requests for attention. We assumed that these models are given. However, in practice, we might need to learn such models through experiments and then design adaptive shared autonomy policies based on the knowledge accumulated over the learning phase. In this respect, a possible solution is to incorporate joint learning and control policy synthesis, for instance, PAC-MDP methods [25], into multi-objective MDPs with temporal logic constraints.

Another limitation in modeling is that the current cognitive model cannot capture all possible influences of the operator's cognition on his performance. In the future work, we aim to extend this modeling framework to capture performance degradation of the operator in the working environment.

APPENDIX

Consider a multiobjective MDP $\mathcal{M} = \langle V, \Sigma, \Delta, \boldsymbol{D}_0, \vec{r}, \gamma \rangle$ where $\vec{r} = (r_1, r_2, \ldots, r_n)$ is a vector of reward functions and γ is the discount factor, let $U_i(\cdot, f_i^*)$ be the vectorial value function optimal for the i-th criterion, specified with the reward function r_i. An approximation of the Nadir point for the i-th criterion is computed as follows, $U_i^N = \sum_{v \in V} \boldsymbol{D}_0(v) \min_{j=1,\ldots,n} U_i(v, f_j^*)$ where $U_i(\cdot, f_j^*)$ is a vector value function obtained by evaluating the optimal policy for the j-th criterion with respect to the i-th reward function. The weight vector after normalization is defined as $\lambda_i = \frac{w_i}{\left|U_i^I - U_i^N\right|}$.

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